 MODULE 5 TEST

1. Lacy has 5 more marbles than Theodore. Eric has one plus twice as many marbles as Theodore. Parametric equations to represent this situation are \( L(T) = T + 5 \) and \( E(T) = 1 + 2T \). Find an expression for the number of marbles Eric has in relationship to Lacy.

   a) \( E(T) = 3T + 6 \)  
   b) \( L(E) = \frac{1}{2}E + 4.5 \)  
   c) \( E(L) = 2L - 9 \)  
   d) \( E(L) = 2L - 4 \)

2. Find the rectangular coordinates for the following set of parametric equations.

   \[
   x = \frac{2}{3} \tan(t) \\
   y = \frac{3}{4} \sec(t)
   \]

   a) \( \frac{16x^2}{9} + \frac{9y^2}{4} = 1 \)  
   b) \( \frac{9y^2}{16} + \frac{4x^2}{9} = 1 \)  
   c) \( \frac{4x^2}{9} - \frac{9y^2}{16} = 1 \)  
   d) \( \frac{16y^2}{9} - \frac{9x^2}{4} = 1 \)

3. Use parametric equations to describe the trajectory of a tennis ball hit 2.5 feet off the ground at an angle of 16° with an initial velocity of 55 feet/sec. (Assume the court does NOT have a net.)

   a) \( x = 55t \cos(16^\circ) \)  
   \( y = 55t \sin(16^\circ) - 16t^2 + 2.5 \)  
   b) \( x = 55t \cos(16^\circ) \)  
   \( y = 55t \sin(16^\circ) \)  
   c) \( x = 55 \cos(16^\circ) - 16t^2 + 2.5 \)  
   \( y = 55 \sin(16^\circ) \)  
   d) \( x = \cos(16^\circ) + 55t \)  
   \( y = \sin(16^\circ) + 55t - 16t^2 + 2.5 \)
4. In the previous problem, how long does it take the ball to hit the ground?
   a) between 57 and 58 seconds  c) between 1.1 and 1.2 seconds
   b) between 1.0 and 1.1 seconds  d) between 0.13 and 0.14 seconds

5. What is the maximum height the ball from problem 3 reaches (to the nearest tenth of a foot)?
   a) 5.4 feet  c) 26.4 feet
   b) 8.6 feet  d) 6.1 feet

6. A ball is thrown straight up from a height of 1 meter above the ground with an initial velocity of 20 meters/second. What parametric equations best represent the path of the ball at time, \( t \)? (velocity of gravity is 4.9 meters/sec)
   a) \[
   y = 1 + 20t - 4.9t^2
   \]
   b) \[
   x = 0, \quad y = 1 + 20t
   \]
   c) \[
   x = 2t, \quad y = 1 + 20t
   \]
   d) \[
   x = 2t, \quad y = 1 + 20t - 16t^2
   \]

7. When does the ball from the previous problem reach its maximum height?
   a) between 1.7 and 1.71 seconds  c) between 2.04 and 2.05 seconds
   b) between 21.406 and 21.407 seconds  d) between 2.26 and 2.27 seconds
8. A golfer drives a ball with an initial velocity of 60 yards per second. The ball is struck at an angle of 47° above the horizontal. When the ball is hit, a 1.47 yard/second wind blows against the ball, pushing it back towards its starting point. Which set of parametric equations best represents the situation? (Velocity of gravity is 5.4 yards/sec)

   a) \[ x = 60t \cos(47°) - 5.4t^2 \]
      \[ y = 60t \sin(47°) - 1.47t + 1.47 \]
   b) \[ x = 60t \cos(47°) \]
      \[ y = 60t \sin(47°) \]
   c) \[ x = 60t \cos(47°) + 1.47 \]
      \[ y = 60t \sin(47°) - 5.4t^2 - 1.47t \]
   d) \[ x = 60t \cos(47°) - 1.47t \]
      \[ y = 60t \sin(47°) - 5.4t^2 \]

9. What is the maximum height the ball from the previous problem reaches?

   a) about 89.1 yards    c) about 160.6 yards
   b) about 4.07 yards    d) about 300 yards

10. Using the parametric equations from problem 8, if the ball had to be hit over a pond that is 20 yards in diameter and whose front edge is 200 yards from where the golfer hit the ball, how far does the ball land from the back side of the pond if it was a straight shot (no hook or slice)?

    a) about 220.5 yards    c) about 1.5 yards
    b) about 100.5 yards    d) the ball fell in the pond

11. What is the center of the circle represented by \[ 3x^2 + 3y^2 - 12x + 15y + 1 = 0 \]?

    a) \( \left( -2, \frac{5}{2} \right) \)    c) \( \left( 4, \frac{25}{4} \right) \)
    b) \( \left( 6, -\frac{15}{2} \right) \)    d) \( \left( 2, -\frac{5}{2} \right) \)
12. A horse walker forces horses connected to it to walk in a circle for exercise. The area of the circle formed is approximately 1257 square feet. If we let the center of the horse walker represent the point (1, 3), which set of parametric equations represents the path of the horses if they walk in a counter-clockwise direction?

a) \( x = \cos(t) + 400 \) \\
\( y = 3 \sin(t) + 400 \)

b) \( x = 3 + 400 \sin(t) \) \\
\( y = 1 + 400 \cos(t) \)

c) \( x = 1 + 20 \cos(t) \) \\
\( y = 3 + 20 \sin(t) \)

d) \( x = 20 + \sin(t) \) \\
\( y = 20 + 3 \cos(t) \)

13. If a parabola opens to the left, and the distance from the vertex to the focus is 2.5 inches, what is the equation representing the parabola if its vertex lies at the point (-3, -4)?

a) \( x + 3 = -\frac{1}{10}(y + 4)^2 \)

b) \( y - 3 = \frac{1}{4}(x - 4)^2 \)

c) \( y + 4 = \frac{1}{10}(x + 3)^2 \)

d) \( \frac{1}{10}(x + 3) = (y + 4)^2 \)

14. Max runs along an elliptical trail. If he runs through the center of the ellipse, the longest distance is 500 yards (in the North/South direction), and the shortest distance is 300 yards (in the East/West direction). Which set of parametric equations represents Max’s path if he runs in a clockwise direction and if the center of the trail is represented by the point (-5, 2)?

a) \( x = 2 + 300 \sin(t) \) \\
\( y = -5 + 500 \cos(t) \)

b) \( x = -5 + 300 \sin(t) \) \\
\( y = 2 + 500 \cos(t) \)

c) \( x = 300 + 2 \sin(t) \) \\
\( y = 500 - 5 \cos(t) \)

d) \( x = -5 + 300 \cos(t) \) \\
\( y = 2 + 500 \sin(t) \)
15. A model suspension bridge has beams that are 200 centimeters apart and rise 55 centimeters above the model’s road. The cable between the beams is connected at the top of each beam and takes on the shape of an upward-facing parabola with a vertex 5 centimeters above the model’s road. What is the equation of the parabola if the vertex of the parabola lies on the y-axis, and the road represents the x-axis?

a) \( x = \frac{1}{200} (y - 5)^2 \)  

b) \( y + 5 = \frac{1}{4} x^2 \)  

c) \( y - 5 = \frac{1}{200} x^2 \)  

d) \( x + 5 = \frac{1}{50} y^2 \)
ANSWERS

1. c
2. d
3. a
4. b
5. d
6. a
7. c
8. d
9. a
10. b
11. d
12. c
13. a
14. b
15. c