Visualizing the Solutions of a Quadratic Equation

Background: Often, students don’t seem to “connect” the ideas of the plot of a parabola and the solution of a quadratic equation. The point of this lesson is to make that connection. Part of making the connection will be to draw a comparison with a linear function, which should be well understood already.

Objectives: The student will understand the relationship between the x-intercepts of the graph of a parabola and finding the solutions of a quadratic equation algebraically.

Materials: Graph paper and pencil
Graphing calculators
Worksheets

Introduction: Start by having the students graph a line: \( y = 2x + 4 \). From the graph, ask them to read off the x-intercept. Next – have them solve for the x-intercept algebraically, by setting \( y = 0 \) and solving.

Procedures: Have the students each create (by hand) a graph of the function:
\[ f(x) = x^2 - 6x + 8 \]
using point-by-point plotting. Some things to discuss while they are doing this are:
- the general shape of the plot will be a parabola
- how to start by finding the vertex
- how to determine which way the parabola will point (up or down)
- how to use the symmetry of the parabola to get additional points

Have them write down the x-intercepts from the graph.
Next, have them solve the equation: \( x^2 - 6x + 8 = 0 \), by factoring.
Compare the solutions of the equation to the x-intercepts which were found on the graph.

Discussion Points:
- Note the parallel between the linear equation and the quadratic equation: in both cases we set \( y = 0 \) and solve for \( x \), although we have to use different techniques to solve the different types of equations.
- Reinforce the idea of “visualizing” the solution – the points where the graph goes through the x-axis are the “zeros” or the “solutions” to the equation.
- Note that the two points which were found are “real-number solutions”, which leads into the next part of the procedure:
Procedure (continued):
Have the students graph:  $y = x^2 - x + 2$ (either by hand or with a graphing calculator)
Discuss: what do they think it means in terms of what the solutions are?
Have them solve the equation:  $x^2 - x + 2 = 0$ using the Quadratic Formula, which has
two imaginary solutions.

Discussion Points:
• Note that the graph shows that the equation has no real-number solutions, because
it has no x-intercepts, but we can still solve algebraically and find the imaginary
solutions.
• What would it mean graphically if an equation has exactly one real-number
solution? Have the students come up with a graph of this, and a corresponding
function.

Assessment/Evaluation:
Have a worksheet available for students to work on in class and finish at home if
necessary.
Suggested content: have a variety of different quadratic equations, where the
students:
• start by plotting the equation (by hand or with a calculator)
• use the plot to predict the number and type of solutions – and the actual solutions
  if possible
• then solve algebraically to (hopefully) confirm their predictions

Extensions: Start with the solutions, and have students write quadratic equations that
fit the solutions. Have them come up with more than one equation per set of
solutions.

Vocabulary:
• parabola
• vertex
• symmetry
• linear, quadratic equations
• solutions, roots
• real numbers, imaginary numbers
• factoring, Quadratic Formula

Time of Lesson: one class period

Tips on Teaching: the graphing/solving connection is KEY, so draw big graphs and
mark them very clearly.