Title: Applications of parabolas

Objectives: Students will use their knowledge of quadratic equations and parabolas to solve problems.

Materials: Meter stick, paper, pencil, worksheets, graphing calculators

Introduction: (Engage and Explore): Some students tend to get anxious about word problems, so begin the lesson by putting the quadratic equation \( y = -4.9t^2 \) on the board. Explain to them that this is the equation of an object in free-fall released from a height of zero, where \( y \) is measured in meters and \( t \) is in seconds. Let students get in pairs and give each pair a meter stick. Explain to them that one student is to hold the meter stick vertically in the air so that the zero end of the stick is about shoulder height. The other student is to take his thumb and index finger and spread them apart about 1.5 inches and place the opening such that the zero edge of the meter stick is the same height as the top of his finger/thumb. Without any notice, the student holding the meter stick lets go and the other person pinches his fingers together and grabs the stick. The reading at the top of the finger is taken from the meter stick. Then the roles are reversed. The students are then asked to convert the measurements to meters and let this value be \( y \) in the equation. Then let them solve for \( t \) (ask them which one is correct, the positive or the negative value and why). Tell them that they have calculated their reaction time! Students generally enjoy this activity a great deal. Let them repeat it several times and talk about the mean and median values. This activity incorporates many great topics.

Procedures: There are many examples of the uses of parabolas in real life applications. Have students call out things they believe are made of parabolas. Examples could be suspension bridges, a slack rope, the trajectory of a ball tossed across the room, the trajectory of a rocket, satellite dishes, head lights, flashlights, parabolic mirrors, Le Four Solaire at Font-Romeur (a parabolic solar reflector), and parabolic reflector heaters. (The links below are excellent resources on applications of parabolas.) Introduce to the class the quadratic equation that governs the height of a projectile: \( y = \frac{1}{2}gt^2 + v_0t + s_0 \) where \( v_0 \) is the initial velocity, \( s_0 \) is the initial height of the object, \( g \) is acceleration due to gravity. Allow students to pick an application and write a report. Guide the students when they are picking their topic to be sure that they are appropriate. If the class has access to a computer, visit the websites listed below and read them as a class.

Some information that students find interesting: Each side of the Golden Gate Bridge (North and South) of the main towers is a parabola. The total distance on
the x-axis of each parabola is 1,125 ft and the change in height on the y-axis is 500 ft. One can be shown as an increasing exponential and the other as a decreasing exponential.

Adaptations: Students often find this an exciting lesson. See if you can bring a guest speaker in for this lesson (math teachers rarely get that opportunity!). Perhaps an engineer or a physicist or anyone who deals with applications of parabolas would be willing to spend a class period talking to the class.

Discussion Questions: After explaining that the trajectory of a golf ball is a parabola, ask the students if it is always a parabola. This is a good time to explain that the parabola is a good mathematical model for the trajectory, but drag, wind and other things can affect the path the ball takes.

Assessment/Evaluation: Students are graded on participation in class and on the report they write.

Extensions: Have students draw a line \( l \) and a point \( P \) not on the line on a piece of paper. Then have them fold and crease the paper so a point \( O \) on the line coincides with \( P \). Have them repeat this process until they can determine the shape of the curve that is being formed. Be sure they use different points \( O \) on the line. Tell them to repeat this investigation using different point \( P \). Ask them how the distance from \( P \) and \( l \) affect the resulting curve?

Suggested Readings: Touger, Jerold Introductory physics, 2005
Barter, James The Golden Gate Bridge, 2001

Links:
http://mathforum.org/te/alejandre/four/parabola.html
http://mathforum.org/mathtools/refresh.html?url=http%3a%2f%2fmathdemos%2fecsu%2eedu%2fparabolane%2fparabola_main%2ehtml
http://www.ima.umn.edu/~arnold/calculus/bounce/bounce1/bounce-g.html
http://www.carondelet.pvt.k12.ca.us/Family/Math/03210/page3.htm

Vocabulary: trajectory, suspension bridge,

Academic Standards
111.32.b. (1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
111.32.b.(2) The student uses the properties and attributes of functions.
111.32.b.(4) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.
111.32.d.(1) The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.
111.32.d.(2) The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

**Time of Lesson:** One 50-minute lesson

**Tips on teaching:** If your school has a physics department, perhaps one of the teachers or some of their students can come to your math class and talk about the applications of parabolas that they are discussing in their classes.