Representing functions

There are a number of methods to represent functions, from the particular to the abstract. Each has some purpose or another. In this brief essay we review some of the types. It is important to remember that a function is a special type of relation between two sets of objects: the domain and the range.

\[
\text{Domain} \longrightarrow \text{Function} \longrightarrow \text{Range}
\]

The restriction is that for each element of the domain there is only one element in the range associated to it by the function.

**Particular Forms**

1. *(Equation form)* The basic form you are using in pre-calculus is the symbolic function notation

   \[ y = f(x) \]

   Here we always mean that for each \( x \) in the domain of \( f(x) \), \( y \) is the corresponding value in the range. We often call the function just by the letter "\( y \)" but not always. For example, \( y = x^2 \) is the familiar equation of a parabola, and here we would say "the function \( y \) is...

2. *(Function form)* In this form we don’t write the equation and refer to the function itself, \( f(x) \). If \( D \) and \( R \) representent the domain and range of the function respectively, then we can write that \( f(D) \subseteq R \). For example, \( f(x) = x^2 \) represents the same function as above. There is no mention of \( y \).

3. *(Data form)* Sometimes the function is discrete in that the domain \( D \) is finite. Let us suppose that \( D = \{x_1, x_2, \ldots, x_n\} \). Then for some function \( f(x) \), we can write the function as a collection of ordered pairs

   \[ \{(x_j, f(x_j)) \}, \ j = 1, \ldots, n \} \]

   For example, consider \( f(x) = x^2 \) and \( D = \{1, 2, 3, 4, 5\} \). Then we write the function as

   \[ \{(1, 1^2), (2, 2^2), (3, 3^2), (4, 4^2), (5, 5^2)\} = \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)\} \]

   Another way to do this is to avoid referencing the function “\( f \)” altogether and writing just a collection of ordered pairs

   \[ \{(x_j, y_j) \}, \ j = 1, \ldots, n \} \]

   Alternatively, we can simply enumerate the pairs

   \[ (x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n) \]

   In the example just above with \( f(x) = x^2 \) and \( D = \{1, 2, 3, 4, 5\} \), the second representation \( \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)\} \) avoids mention of the function altogether.

   Note the use of the “dots” notation to fill in or enumerate a list of undetermined size. This form \( \{(x_j, y_j) \}, \ j = 1, \ldots, n \} \) is critically important when the function — as a formula — is not known but only its values at select points are known. That is to say, there is NO formula given. Real life data often comes this way. There is, for example, a
function relation between the population and the number of high school students for each country. But there is no formula. Again, there is a function that gives the total oil pumped from a well at each given instant during the day, but there is no formula, only sampled data.

4. *(Tabular form)* This form is similar to the form above except the data is arranged in a table as follows.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$y_1$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$y_2$</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$y_3$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$x_n$</td>
<td>$y_n$</td>
</tr>
</tbody>
</table>

The parabola example looks like this.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

5. *(Graphical form)* One certain way to show functions is to draw a picture of one. In two dimensions this is always easy and effective. However, the student needs to be quite familiar with the representation of information on a chart or graph.

6. *(Domain-Range form)* In advanced mathematics courses, where the domain and range are very important, a common notation to represent a function is
It is important to understand that this method, while very, very intuitive does not convey exact information about the function, the domain, or the range. Some examples can do this, but in general, this is not so.

**The symbolic visual form**

While you might consider the first two forms above as symbolic, we mean the symbolic form to imply the visual or *mapping* idea of a function. It is designed to show how a function conveys values from one set to another set with the particular rule. Notice in all of the forms, there are no numbers (except for axis labels) and essentially no symbolic functions, just letters.

1. *(Machine form)* The machine form for a function makes it look like some sort of blender or machine that processes elements of the domain into elements of the range.

   ![Diagram of a function](image)

   Because it is customary to regard a machine as a device that always does the same thing to the same input, this form is very useful to show students the idea of a function and to emphasize that for each \( x \), there is just one function value \( f(x) \).

2. *(Mapping form)* The mapping form of a function emphasizes the set nature of the domain and range and illustrates that the function conveys, or maps, one set to the other.

   ![Diagram of a function](image)

   Here we have *visualized* the domain and range as distinct objects, even though they are
often the same set of numbers.

3. *(Domain-Range form)* In advanced mathematics courses, where the domain $D$ and range $R$ are very important, a common notation to represent a function is

$$f : D \rightarrow R$$

For example, a function with domain and range equal to the reals $R_1 = (-\infty, \infty)$ is written as $f : R_1 \rightarrow R_1$. In this way we can express all functions with the given domain and given range. This takes us a little far afield, but mathematicians and scientists often talk about entire classes of functions. Classes of functions considered in pre-calculus are polynomials, continuous functions, and piecewise continuous functions. It is best to note that some students have great difficulty imagining and working with entire classes of objects.

4. *(Graphical method)* This method is much like the previous graphical method with even less information conveyed. A typical example is shown by the following graph.

![Graphical representation](image)

Here, there is no designation of domain, range, or anything really about the function. Such representations are useful for describing properties of the function, such as where it increases, decreases, peaks out, bottoms out, and so forth. When describing such concepts, domain, range, and numbers often get in the way. Students should be very familiar with graphing functions, for often they think in specifics and become confused when specifics are missing, as in the above illustration.