## Resources for pre-calculus - II

A Teacher Quality Grant Project supported by the THECB and TEA

### Resources on algebra and pre-calculus

( with an emphasis on web-based interactives)

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### Real Numbers

- **MathSteps** - lesson ideas for teaching real number classifications
- **Purplemath** - a nice lesson on number types
- **Multiple Choice Test of Number Sets** - a nice interactive test over number sets definitions and closure under the various sets - would make a great review or extra credit
- **Types of Numbers** - a great 3D Venn of the number sets - you will have to download a plug-in to see it

### Relations & Functions

- **Vertical Line Test** - a Project Interactive applet allowing students to investigate the vertical line test
- **Possible or Not?** - a Project Interactive applet - students decide if a given graph is a function and if it represents a possible situation in the real world
- **Purplemath** - a wealth of lessons including function definition, function notation, function transformations, direct and inverse variation, etc.
- **Moving Man** - a great applet by Lisa Denise Murphy - drag the “moving man” and the applet generates distance, velocity, and acceleration graphs
- **Operations on Functions** - a graphing applet that will graph \( f(x) \), \( g(x) \), and the sum, difference, product, quotient, inverse, and composite functions - a little slow to load, but worth it - includes linear and trig functions
- **Graph or Not a Graph?** - an interactive test - for each graph click “function” or “not a function” then check out your score - a great review or extra credit activity
- **Function Transformation Applets** - separate applets for vertical translations, horizontal translations, vertical stretch, and horizontal stretch
- **Function Composition Applet** - input \( f(x) \) and \( g(x) \) then drag a point to vary the input - watch the composition change accordingly
- **Inverse Functions** - an explanation of inverse functions with very nice visuals

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file://C:\projects\math_homepage\precalculus\Precalculus_home\resources\resources2.htm  4/14/2004
Exploring Inverse Functions - a very nice treatment of inverse functions including Java applets and a QuickTime movie TOP

Linear Functions
Function Toolkit - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
Modeling Orbital Debris Problems - uses the orbital debris problem to investigate linear and quadratic functions - with printable student activity sheets
Linear Regression & Correlation - i-Maths Investigation - investigate best fit line with an interactive online applet, online and downloadable student questions
Exploring Linear Functions - a nice applet showing the effect of changes to m and b on the graph of y = mx + b
Exploring Linear Data - printable student pages investigating four different situations modeled by linear functions - high student interest topics
Fun and Sun Rent-a-Car - an exploration by Cynthia Lanius modeling various rate plans using a Java based grapher - also available as a print version
Function Machine - from Utah State University - a very nice animated function machine - students look for patterns and predict results
Slope Slider and Function Flyer - Project Interactive applets designed for student exploration - very nice
Function Machines from Project Interactive - several to choose from
MathSteps - lesson ideas for teaching linear equations
Purplemath - a wealth of lessons including all aspects of linear functions
The Big Function Graph Puzzle - earn points by correctly matching the equation to the graph - a great way to review transformations - combination of linear and quadratic functions
Linear Functions Applets - Java applets for slope-intercept, point-slope, and standard form - change the values of the coefficients and watch the resulting change in the graph TOP

Systems
Systems of Equations in Electronics - applications of systems to electronics, several solving methods with interactive "solvers", includes matrices
Supply and Demand - An Application of Linear Systems - three printable student activity sheets with internet extensions
Purplemath - a wealth of lessons including systems of equations and inequalities - includes solving by Gaussian elimination and systems word problems
Solving Two Equations - from Webmath - the users inputs two equations - the applet solves the system and walks the user step by step through the process
Intersection Calculator - input the slope and y-intercept of two lines and the calculator finds the intersection

TOP

Quadratic Functions
Exploring Parabolas - a JavaSketchpad interactive
Function Toolkit - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
Completing the Square Applet - students change the parameters and the calculator completes the square
Purplemath - a wealth of lessons including quadratic formula, completing the square, etc.
The Big Function Graph Puzzle - earn points by correctly matching the equation to the graph - a great way to review transformations - combination of linear and quadratic functions
Where does the quadratic formula come from? - a nice derivation of the quadratic formula that
would be good for extra credit or inclusion in a webquest

**Quadratic Solver** - input the values of the coefficients and the solver gives the discriminant and the roots (including complex roots)

**TOP**

**Polynomial Functions**
- **Function Toolkit** - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
- **Polynomial Roots Calculator** - calculates the roots of a polynomial function up to degree 5 (real roots only)
- **Polynomial Function Graphs** - Java applets for graphing polynomial functions up to the fifth degree - change the values of the coefficients and watch the resulting change in the graph

**TOP**

**Rational Functions**
- **Whelk-Come to Mathematics: Using Rational Functions to Investigate the Behavior of Northwestern Crows** - an i-Maths Investigation - collect data in a lab setting or use online data, analyze with online tools - very nice online student version
- **Purplemath** - a wealth of lessons including rational functions and complex fractions
- **Transformations on 1/x** - an interactive applet allows the user to change parameters and watch the effect of the graph - very nice
- **Rational Functions** - more material from the makers of the above applet - several other applets also available as well as good notes
- **Precalculus On-line Lab on Rational Functions** - applet allows for graphing multiple functions in various colors - exercises lead students from transformations on 1/x to slant asymptotes and partial fractions - first part could easily work in Algebra
- **Rational Functions Match-up Puzzles** - two interactive puzzles matching graphs of rational functions to their equations - fun
- **Transformations of \( f(x)=1/x \)** - Java applets for graphing 1st degree rational functions - change the values of the coefficients and watch the resulting change in the graph
- **Rational Function Applets** - Java applets for graphing rational functions up to the 3rd degree - change the values of the coefficients and watch the resulting change in the graph

**TOP**

**Radicals and Roots and Square Root Functions**
- **Function Toolkit** - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
- **Purplemath** - a wealth of lessons including radicals
- **Radical Equations** - a nice step by step tutorial - would be great for the student who was absent the day you taught this!

**TOP**

**Exponential Functions**
- **Function Toolkit** - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
- **Chickscope EggMath** - investigate the value of \( e \) by studying chicken embryo growth rate - a nice site
- **Shedding Light on the Subject: Function Models of Light Decay** - an i-Maths Investigation - collect data in a lab setting or use online data, analyze with online tools - very nice online student version
- **Purplemath** - a wealth of lessons including exponential functions
- **Exponentials Resources from Charles Sturt University** - includes slide shows, teaching ideas,
tutorials, applications, assessments

In Search of the Missing e - a webquest that review exponential equations and investigates various applications

Population Growth - an internet project using real census data to study population growth

Transformations of $f(x) = 2^x$ - Java applets for graphing exponential functions - change the values of the coefficients and watch the resulting change in the graph

Financial Applications of Exponential Functions - explore various applications of exponential functions - each with each own specialized calculator

Logarithms

Function Toolkit - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes

Purplemath - a wealth of lessons including logarithms

Logarithm Resources from Charles Sturt University - includes slide shows, teaching ideas, tutorials, applications, assessments

Matrices

Purplemath - a wealth of lessons including lots of matrices

Stepwise Matrix Multiplier - an interactive applet showing step by step how to multiply matrices - very nice

Matrix Multiplier - the next step from the previous source - students must perform the calculation and the applet checks their work

Virtual Manipulatives

Virtual Algebra Tiles - from Utah State University - my favorite web-based tiles - easy to manipulate - includes activities

Algebra Balance Scales - from Utah State University - place $x$ and units blocks on a scale and keep it balanced while solving for $x$ - very nice - includes lots of problems to solve

Expression Balance - interactive pan balance allows numeric or algebraic expressions to be entered and compared - great for deciding if expressions are equal

Parametrics

Plotting Parametric Curves - a graphing utility for parametric curves

Parametric Equations from Visual Calculus - a nice tutorial on plotting parametric equations with short flash animations

Another Parametric Graphing Applet - shows various curves and challenges students to graph them

Polar Coordinates and Equations

Polar/Rectangular Coordinates Converter - a nice applet that helps students understand the difference between point forms

Polar/Cartesian Coordinates Applets - students drag the points and watch the coordinates change to gain an intuitive understanding of the coordinate systems

Polar Curves - short movies showing the plotting of polar curves - very nice

Conic Sections

Exploring Parabolas - A JavaSketchpad Interactive connecting directrix and focus to the parabola’s
Exploring Ellipses and Hyperbolas - A JavaSketchpad Interactive
Chickscope EggMath - uses egg shape to compare ellipses and ovals - a nice site
Conic Flyer - a Project Interactive applet - change the parameters and watch the effect on the graphs of various conics - would make a great introduction
Purplemath - a wealth of lessons including conics
Sections of a Cone Applet - an applet that allows the student to set parameters for a plane and see how the resulting plane intersects a cone
Conics by Xah Lee - a wealth of information from history to equations to applications and more links - beautiful Mathematica created graphs
An Introduction To Conic Sections - by James Sellers - an online textbook with good examples - includes rotation of axes
Parabolic, Elliptical, and Hyperbolic Reflectors - QuickTime movies of a pulse reflecting off the surfaces of conic reflectors - great!
Conics Notes & Exercizes - by the author of the above movies - a very good resource
Conic Sections at Zona Land - very nice visuals of the plane cutting the cone to form the various conics

Trigonometric Functions
Function Toolkit - A JavaSketchpad Interactive - drag a point along the x-axis and generate a linear function - see effects of parameter changes
Sine and Tangent Curve Animations - QuickTime movies - very good
Trigonometric Functions Resources from Charles Sturt University - includes slide shows, teaching ideas, tutorials, applications, assessments
Operations on Functions - a graphing applet that will graph \( f(x) \), \( g(x) \), and the sum, difference, product, quotient, inverse, and composite functions - a little slow to load, but worth it - includes linear and trig functions
Trig Graphs from the Unit Circle - a Java applet that shows how the sine, cosine, and tangent graphs are generated from the unit circle
Trig Graphs Match-up Puzzles - two interactive puzzles matching graphs of rational function to their equations - fun - from the same source as the link above
Sound Sketch - see how sine waves convert to sound - sketch and sound and then listen to see how varying the wave varies pitch and intensity - fun!
Sine Wave Applet - set the frequency and amplitude of two sine waves then watch the applet graph both waves and their sum
Superposition Applet - type in any functions \( f(x,t) \) and \( g(x,t) \) and see their sum - really a Physics site with great flexibility
Common Angles Around a Circle - shows all the common angles in the unit circle and gives their measure in degrees and radians
Trig Functions Point Definitions - a really nice applet that shows a point on the x-y plane with its accompanying reference triangle and the point's corresponding position on the graph of the trig function - all six trig functions available - one of my favorites!
The Radian - a great definition of the radian with a Java animation showing the radius equal to the arc length

Trigonometric Identities
Identities Applets - four Java applet quizzes over identities and a true/false quiz

TOP
Other Trig Topics
Trigonometry Resources from Charles Sturt University - includes slide shows, teaching ideas, tutorials, applications, assessments
Navigational Vectors - internet project that uses real time navigational data in the study of vectors - fun!
Car Storm Chaser - a single vector controls the direction of your storm chaser car - follow the storm
Airplane Storm Chaser - two vectors control the motion of your airplane - follow the storm
Resultant of Forces - a very nice applet for teaching vector sums
Degrees/Minutes/Seconds Calculator - convert degrees to DMS and vice versa
Right Triangle Solvers - a trio of solvers - input the two known sides and let the solver find the third

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Probability and Combinatorics
Frequency Distribution Calculator - calculates the frequency distribution of possible sums from throwing multiple dice and possible outcomes from tossing a coin and graphs the results
Virtual Coin Toss - from Utah State University - a very nice interface - table and graph of data from multiple coin tosses
Project Interactive - a whole range of probability simulation applets - my favorites are "Rabbits and Wolves", "Life", and "Buffon's Needle"
Cut-the-Knot Probability Page - a collection of 20+ problems in probability - most are interactive Java applets
Plinko! and Probability - vry high interest for students - includes online simulators and teacher notes
The Amazing Mathematical Object Factory - generates objects for combinations, permutations, etc.
The Combinatorial Object Server - the more sophisticated cousin of the site above - this one worked better for me
Thoughts on Teaching Permutations, Combinations and the Binomial Theorem - interesting
Binomial Theorem - good information here
Mrs. Glosser's Probability Lessons - interactive learning modules - some available on CD only
What are the Odds? - a collection of lesson plans on probability with an interactive quiz that helps to reveal student misconceptions.
A Casino Game - play a roulette type game and explore the math behind it
Three Easy Pieces - Java applets let you explore the birthday problem, the poker problem, and Bufoon's needle problem
The Josephus Problem - play an elimination game then study the math to move from seeing a pattern to explaining the pattern using recursive reasoning

Mathematical Induction
Mathematical Induction - good information - could supplement your text

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Series and Sequence
Visualizing an Infinite Series - a great tool for understanding infinite series from Cynthia Lanius
Sum of Arithmetical Progression - an interactive applet leading the derivation of the sum formula
Series & Sequences Resources from Charles Sturt University - includes slide shows, teaching ideas, tutorials, applications, assessments
Numerical Computation Tools for Sequence and Series - input the formula for the nth term and quickly computer terms in the sequence or partial sums - quick and easy to see terms and sums approaching a limit
Arithmetic or Geometric? - an interactive quiz - label each sequence as arithmetic or geometric then check out your score - would make a great review or extra credit
**Convergent or Divergent?** - an interactive quiz - label each sequence as convergent or divergent then check out your score - would make a great review or extra credit

**Continuity, Limits, and Tangents**

**Stressed Out: Slope as a Rate of Change** - Cynthia Lanius investigates the slope of a tangent to a parabola

**Moving Man** - a great applet by Lisa Denise Murphy - drag the "moving man" and the applet generates distance, velocity, and acceleration graphs - could be used to introduce derivatives

**Numerical Computation Tools for Sequence and Series** - input the formula for the nth term and quickly computer terms in the sequence or partial sums - quick and easy to see terms and sums approaching a limit

**Continuous or Discontinuous?** - an interactive quiz - label each sequence as continuous or discontinuous then check out your score - would make a great review or extra credit

**Data Sources, Scatterplots and Regressions**

**Vital Signs Data and Spreadsheet Analysis** - Data collection and analysis activity - build your own spreadsheets from the online data or use the online examples

**Brrrr....Wind Chill is Chilly** - finding the line of best fit for wind chill data - a very nice investigation with both math and science extensions

**Put the Heart into Mathematics: Cardiac Output, Rates of Change and Accumulation** - an i-Maths Investigation - collect data in a lab setting and analyze with online tools - can be used from Algebra 2 to Calculus

**Using Graphs, Equations, and Tables to Investigate the Elimination of Medicine from the Body: Modeling the Situation** - use an interactive applet to collect data, use online tools to investigate the effects of parameter changes

**Real Data Sources from ENC** - lots of links to sources for real-life data

**Virtual Scatterplotter** - from Utah State University - plot data points and watch the best-fit line adjust accordingly - really nice!

**Flowing Through Mathematics** - interactive applet simulates water flowing from a tube through a hole in the bottom - adjust the diameter of the hole and gather data for the height or volume of water in the tube - great activity questions!

**Picturing Functions** - click on the picture (10 to choose from) to collect data points for the curves - the applet plots the data, performs a regression, and graphs the curve over original picture then calculates the arc length of the curve - awesome!

**Graphing Calculators**

**Virtual Grapher** - from Utah State University - a very user friendly interface - graph up to 3 functions in 3 colors, change parameters with sliders

**Graph Sketcher** - a Project Interactive applet - type in your function, set your view window and click graph - a nice source for printing graphs

**SimCalc** - software for the TI-83 plus allowing it to run simulations - sample available for download

**Simple Data Grapher** - input the legend and scale for your graph then input the data - the grapher creates a nice printable graph

**EZ Graph** - a fully functional online grapher

**TI Tutorials** - how to documents for the TI-82 and TI-83

**Casio Programs** - downloadable programs for the Casio 9850+

**Casio Programs** - more downloadable programs for the Casio 9850+ and 7400G

**Casio Programs** - more downloadable programs for the Casio 9850+

**Casio Programs** - downloadable programs for the Casio FX 2.0

**Casio Programs** - downloadable programs for the Casio EA100 Data Collector

**ticalc.org** - TI calculator help, downloadable programs, programming contests, etc.
TI Program Archives - downloadable programs for all TI graphing calculators
TI Programs - downloadable programs for the TI CBL Data Collector

Geometer's Sketchpad in Algebra & Precalculus
Exploring Parabolas - A JavaSketchpad interactive with downloadable Sketchpad file
Exploring Ellipses and Hyperbolas - A JavaSketchpad Interactive with downloadable Sketchpad file
Function Toolkit - A JavaSketchpad Interactive modeling all the standard Algebra and Precalculus functions with downloadable Sketchpad files
Conic Sections As the Locus of Perpendicular Bisectors - a JavaSketchpad applet mimicking traditional paper folding of conics - very nice
Least Squares - a very nice JavaSketchpad applet illustrating the least squares regression - explains clearly why its called "least squares"
Sine Waver - another nice JavaSketchpad applet - relates the sine wave to the unit circle
Lissajous Curves - a JavaSketchpad applet showing the family of Lissajous curves
The Witch of Agnesi - a JavaSketchpad applet demonstrating this classic curve

Standardized Tests and Test Preparation
Released TAKS Tests - released versions of the Texas Assessment of Knowledge and Skills - the grade 9-11 tests cover Algebra 1 and Geometry
California State Testing - links to various components of California's testing system
National Assessment of Educational Progress - explore and print questions from the mathematics portion of the NAEP
New York State Regents Exams - archive of past regents exams in pdf format
Virginia State Practice Tests - choose 10, 20, or 40 questions in various subjects and grade levels
Third International Math and Science Survey - grade 12 math and science test

Lists of Famous Curves
Famous Curves Index - from the University of St. Andrews, Scotland - everything from lines to conics and polars to curves you may never have heard of before!
2dcurves - A collection of 674 mathematical curves by Jan Wassenaar with Java interactives - a great site
Visual Dictionary of Famous Plane Curves - by Xah Lee - galleries of curve families - nicely done

History of Mathematics
History Topics Index - from the University of St. Andrews, Scotland - absolutely the best overall site for Math History
Ptolemy's Table of Chords - a great background to trig units, explains Ptolemy's use of double and half angle identities
Earliest Use of Math Symbols - a handy reference for the first use of common math symbols by Jeff Miller
Earliest Use of Math Words - another handy reference tool by Jeff Miller

General Sites
WebMath - a good student tutorial from basic Algebra to Calculus
Purplemath - extensive lessons covering all of High School Math
Illuminations - a collaboration between NCTM and MarcoPolo Education - a great collection of lesson
plans and interactives

_Zona Land_ - lots of great explanations and interactives for Math and Science [TOP]

compiled by De-Vonna Clark